As outlined in the Appendix,

$$\gamma_i^2 + \gamma_i'^2 = 1 - \beta_i^2, \qquad \gamma_i\gamma_j + \gamma_i'\gamma_j' = -\beta_i\beta_j, \tag{6}$$

where β_i represents the direction cosines of the normal to the plane. Substitution of these relations into Eq. (5) and application of the normality condition $\Sigma_i \alpha_i^2 = 1$ give

$$E_\sigma = \frac{3}{2}\sigma\lambda_{100} - \frac{3}{2}\sigma\lambda_{100}\Sigma_i\alpha_i^2\beta_i^2 - 3\sigma\lambda_{111}\sum_{i<j}\alpha_i\alpha_j\beta_i\beta_j . \tag{7}$$

When Eq. (7) is compared with Eq. (5), it is evident that a planar stress of magnitude σ produces essentially the same expression for E_σ as a uniaxial stress along the normal, with the same magnitude but opposite sign. The only difference is an isotropic term $\frac{3}{2}\sigma\lambda_{100}$.

The shape anisotropy energy arises from the demagnetizing effects of magnetic poles. For a thin film of magnetization $4\pi M$, the shape energy is given by (8)

$$E_S = 2\pi M^2\cos^2\zeta , \tag{8}$$

where ζ is the angle between $4\pi M$ and the normal to the plane. Of particular interest in this work are the principal crystallographic planes, the {001}, {110}, and {111} families, and the relations between $\cos\zeta$, the direction cosines α_i of $4\pi M$, and β_i of the normal may be determined from the standard relation

$$\cos\zeta = \alpha_1\beta_1 + \alpha_2\beta_2 + \alpha_3\beta_3 . \tag{9}$$

Thus, it may be readily shown that

$$\cos^2\zeta = \cos^2\theta \tag{10}$$

$$\cos^2\zeta = \frac{1}{2}\sin^2\theta + \sin^2\theta\sin\phi\cos\phi \tag{11}$$

$$\cos^2\zeta = \frac{1}{3} + \frac{2}{3}\sin^2\theta\sin\phi\cos\phi + \frac{2}{3}\sin\theta\cos\theta(\sin\phi + \cos\phi) \tag{12}$$

for the (001), (110) and (111) planes, respectively.

For stress in the (001) plane, $\beta_1 = \beta_2 = 0$, $\beta_3 = 1$ and

$$E_\sigma^{001} = \frac{3}{2}\sigma\lambda_{100}(1-\alpha_3^2). \tag{13}$$

In spherical polar coordinates, Eq. (13) becomes

$$E_{\sigma}^{001} = \frac{3}{2}\sigma\lambda_{100}\sin^2\theta. \quad (14)$$

It is convenient at this point to define $E_{\sigma S} = E_{\sigma} + E_S$ and combine Eqs. (14), (8) and (10), with the result that

$$E_{\sigma S}^{001} = \frac{3}{2}(\sigma\lambda_{100} - \frac{4\pi M^2}{3})\sin^2\theta + 2\pi M^2. \quad (15)$$

For stress in the (110) plane, $\beta_1 = \beta_2 = 1/\sqrt{2}$, $\beta_3 = 0$ and the combination of Eqs. (7), (8) and (11) expressed in polar coordinates yields

$$E_{\sigma S}^{110} = \frac{3}{2}\sigma\lambda_{100} - \frac{3}{4}(\sigma\lambda_{100} - \frac{4}{3}\pi M^2)\sin^2\theta - \frac{3}{2}(\sigma\lambda_{111} - \frac{4}{3}\pi M^2)\sin^2\theta\sin\phi\cos\phi. \quad (16)$$

For stress in the (111) plane, $\beta_1 = \beta_2 = \beta_3 = 1/\sqrt{3}$ and the combination of Eqs. (7), and (8) and (12) expressed in polar coordinates yields

$$E_{\sigma S}^{111} = \sigma\lambda_{100} - (\sigma\lambda_{111} - \frac{4}{3}\pi M^2)[\sin^2\theta\sin\phi\cos\phi + \sin\theta\cos\theta(\sin\phi + \cos\phi)] + \frac{2}{3}\pi M^2. \quad (17)$$

In the analysis reported previously (4), the stress terms of Eqs. (15), (16), and (17) were obtained by applying Eq. (5) directly after selecting suitable sets of γ_i and γ_i' for the three specific cases. It should be pointed out that a different sign convention for σ was used in that work.

Since each of the above equations may be added to Eq. (3) to form the total E, the energy extrema may be determined in the usual manner from $\partial E/\partial\theta = 0$ and $\partial E/\partial\phi = 0$. As in the problem of determining the effects of stress on remanence ratios (3), the major attention is focused on the movement of these axes of extreme energy, i.e., the easy and hard axes of magnetization.

Uniaxial anisotropy induced from the application of stress will be defined as any situation in which the principal easy axis is directed along the normal and all other major extrema are in the plane. In the following section, the relations that control the movement of the pertinent extrema are given, and the conditions for stress-induced uniaxial anisotropy are determined for the specific cases of interest.

Results

For the simplest case of σ in the (001) plane, the energy extrema along the three ⟨100⟩ axes do not rotate. However, the extrema initially (if shape anisotropy is neglected) along the ⟨111⟩ axes may be rotated either towards the normal or into the plane as depicted in Fig. 1. Although only one